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## Unit I – The Structure of Mathematics

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*Part A – Mathematics as a Language*

# LESSON 1 Mathematical Parts of Speech

**Objective:** To identify the five parts of mathematical speech and understand their use.

**Important Terms:**

**Number Symbols** – more commonly called ***numerals***, symbols used to represent quantities.

**Operation Symbols** – symbols such as + (add), – (subtract), × (multiply) and ÷ (divide), used to indicate ***action***.

**Relation Symbols** – symbols such as = (is equal to), > (is greater than), and < (is less than), used to show ***comparisons***, as well as ***combinations*** such as ≥ (is greater than or equal to), ≤ (is less than or equal to), and the negations of all of these (≠, ≯, ≮, ≰), among others.

**Grouping Symbols** – symbols such as ( ) (parentheses), [ ] (brackets), and { } (braces), used to show ***groupings***.

**Placeholder Symbols** – more commonly called ***variables***, symbols such as *a, b, c* (letters of the alphabet) and ❒ (empty boxes), used to hold the place of a number until the number has been identified.

***Example 1:*** Tell which of the following are relation symbols:

**a.** $17\frac{1}{2}$ **b.** ≠ **c.** > **d.** ÷

***Solution:***

**a.** $17\frac{1}{2}$ is not a relation symbol. It is a number symbol meaning seventeen and one-half.

**b.** ≠ is a relation symbol meaning "is ***not*** equal to."

**c.** > is a relation symbol meaning "is greater than."

**d.** ÷ is not a relation symbol. It is an operation symbol meaning "divide."

## Lesson 1 – Exercises:

Tell what part of mathematical speech each of the following is, and state in words what each means.

| | | | |
|---|---|---|---|
| **1.** $c$ | **2.** [ ] | **3.** $3.\overline{45}$ | **4.** + |
| **5.** = | **6.** 63,231 | **7.** $\geq$ | **8.** $\sqrt{5}$ |
| **9.** – | **10.** { } | **11.** $10^4$ | **12.** ÷ |
| **13.** $\not<$ | **14.** ( ) | **15.** $\frac{2}{3}$ | **16.** 41.765 |
| **17.** × | **18.** $n$ | **19.** $\neq$ | **20.** · |

*Part A – Mathematics as a Language*

# LESSON 2 Mathematical Expressions

**Objective:** To identify the four types of mathematical expressions and understand what to do with each.

**Important Terms:**

**Closed Phrase** – any mathematical expression which contains neither a relation symbol nor a placeholder symbol. For example, $2(5 - 1)$ is a closed phrase.

**Open Phrase** – any mathematical expression which contains a placeholder symbol, but does not contain a relation symbol. For example, $6 + [3 - n]$ is an open phrase.

**Closed Sentence** – any mathematical expression which contains a relation symbol, but does not contain a placeholder symbol. For example, $7(4 - 2) = 13$ is a closed sentence.

**Open Sentence** – any mathematical expression which contains both a relation symbol and a placeholder symbol. For example, $9 + 2[n - 4] > 12$ is an open sentence.

***Example 1:*** Identify the following mathematical expressions by looking for relation symbols and placeholder symbols.

**a.** $17 - x$ **b.** $16 - (2 \cdot 5) \geq 6$ **c.** $\frac{12+3}{2}$ **d.** $20 = 4(n - 1)$

***Solution:***

**a.** **Open Phrase** – Because there is no relation symbol, this expression is just a phrase. But there is a placeholder symbol, so the expression is open.

**b.** **Closed Sentence** – Because there is a relation symbol, this expression is a sentence. However, there is no placeholder symbol, so the expression is closed.

**c.** **Closed Phrase** – Because there is no relation symbol, this expression is just a phrase. Neither is there a placeholder symbol, so the expression is closed.

**d.** **Open Sentence** – There is a relation symbol, so the expression is a sentence. There is also a placeholder symbol, so the expression is open.

***Example 2:*** Take the appropriate action for each of the following expressions. For open phrases, use a domain of {0, 1, 2}. For open sentences, use a replacement set of {4, 5, 6}.

**a.** $17 - x$ **b.** $16 - (2 \cdot 5) \geq 6$ **c.** $\frac{12+3}{2}$ **d.** $20 = 4(n - 1)$

***Solution:***

**a.** Since this is an open phrase, the appropriate action is substitution from a domain and evaluation to a range.

Using {0, 1, 2}, we substitute and evaluate:

$$17 - (0) \rightarrow 17$$
$$17 - (1) \rightarrow 16$$
$$17 - (2) \rightarrow 15$$

So the range is {17, 16, 15}.

***Example 2 cont'd:***

**b.** This expression is a closed sentence, so what we can do is tell if it is true or false.

$$\begin{aligned} 16-(2\cdot 5) &\geq 6 \\ 16-10 &\geq 6 \\ 6 &\geq 6 \end{aligned}$$

Since 6 = 6, and that is one acceptable condition, the expression is true.

**c.** Since this is a closed phrase, what we can do is evaluate it.

$$\frac{12+3}{2} \text{ or } \frac{15}{2} \text{ or } 7\tfrac{1}{2}$$

Its value is $7\frac{1}{2}$.

**d.** This is an open sentence, so the appropriate action is to substitute from a replacement set and obtain a solution set of values that make the expression true. Using {4, 5, 6} we substitute and determine truth or falsehood.

| | | |
|---|---|---|
| $20 = 4([4]-1)$ | $20 = 4([5]-1)$ | $20 = 4([6]-1)$ |
| $= 4(3)$ | $= 4(4)$ | $= 4(5)$ |
| $= 12$ | $= 16$ | $= 20$ |
| FALSE | FALSE | TRUE |

So our solution set is {6}.

## Lesson 2 – Exercises:

Tell whether each of the following expressions is an open phrase, closed phrase, open sentence, or closed sentence.

**1.** $6+8(3)$

**2.** $m+2$

**3.** $5-z=2$

**4.** $8-2^3=4-2$

**5.** $\frac{12+8}{5}>2\cdot 2$

**6.** $w^2-w$

**7.** $9y$

**8.** $\frac{15+18}{3}\neq 5+6$

**9.** $3k+11<17$

**10.** $2\cdot 4-1$

In the following exercises, take the appropriate mathematical action with each expression. For open phrases, use a domain of {0, 1, 2}. For open sentences, use a replacement set of {4, 5, 6}.

**11.** $6+8(3)$ **12.** $m+2$ **13.** $5-z=2$

**14.** $8-2^3=4-2$ **15.** $\frac{12+8}{5}>2\cdot 2$ **16.** $w^2-w$

**17.** $9y$ **18.** $\frac{15+18}{3}\neq 5+6$ **19.** $3k+11<17$

**20.** $2\cdot 4-1$

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*Part A – Mathematics as a Language*

## LESSON 3 Translation of Mathematical Symbols

**Objective:** To translate English expressions into expressions with mathematical symbols.

***Example 1:*** Translate the following English phrases into phrases with mathematical symbols.

**a.** The sum of $m$ and 9, decreased by the product of 6 and $y$.

**b.** The quotient of the cube of $x$ and the square of $z$.

***Solution:***

**a.** $\underbrace{\text{the sum of } m \text{ and } 9}_{(m+9)}\ \underbrace{\text{decreased by}}_{-}\ \underbrace{\text{the product of 6 and } y}_{6\cdot y}$

So $(m+9)-6y$ is the desired phrase.

**b.** $\underbrace{\text{the quotient of}}\ \underbrace{\text{the cube of } x}_{x^3}\ \underbrace{\text{and}}_{\div}\ \underbrace{\text{the square of } z}_{z^2}$

So $x^3\div z^2$ or $\frac{x^3}{z^2}$ is the desired phrase.

**Example 2:** Translate the following English sentences into sentences with mathematical symbols.

**a.** Six less than twice a number is 14.

**b.** 58 is greater than the sum of 5 times a number and 18.

**Solution:**

**a.** $\underbrace{\text{six less than}}_{-6}\ \underbrace{\text{twice a number}}_{2n}\ \underbrace{\text{is}}_{=}\ \underbrace{14}_{14}$

So $2n - 6 = 14$ is the desired sentence.

Note that "six less than" means "subtract 6," so this must be placed to the **right** of $2n$.

**b.** $\underbrace{58}_{58}\ \underbrace{\text{is greater than}}_{>}\ \underbrace{\text{the sum of}}_{}\ \underbrace{\text{5 times a number}}_{5m}\ \underbrace{\text{and}}_{+}\ \underbrace{18}_{18}$

So $58 > 5m + 18$ is the desired sentence.

## Lesson 3 – Exercises:

Translate the following English expressions into Mathematics expressions.

1. The sum of $q$ and 6, increased by 4.
2. The product of $m$ and $n$, divided by 11.
3. 27 more than the total of $x$ and $y$.
4. The difference between $p$ and 8.
5. The sum of $a$ and $b$, divided by 6.
6. $a$ to the power of 6, increased by 12.
7. The product of 10 and $z$, decreased by the product of 8 and $x$
8. $y$ multiplied by 6, increased by the product of 5 and $z$.
9. The cube of $y$, subtracted from the product of 5 and $y$.
10. The quantity $x$ plus 5, times the quantity $x$ minus 8.
11. The square of the difference of $y$ and 9, divided by the product of 4 and $t$.
12. The difference between $c$ and $d$, subtracted from the sum of $q$ and $r$.
13. The sum of 6 and $b$, combined with the total of 7 and $c$.
14. The difference between the cube of $v$ and the cube of $w$, divided by the square of $e$.
15. The square of $h$, divided by $b$ less than the cube of $j$.
16. 9 more than 4 times a number is equal to 55.

**17.** The result of 8 times a number, decreased by 20 is the same as the sum of 4 times the number and 20.

**18.** Twice a number is 28.

**19.** 6 times a number, decreased by 12 equals the sum of 3 times the number and 8.

**20.** 150 is the same as 6 times a number decreased by 30.

**21.** The difference between 25, and 4 times a number is less than 100 more than 3 times the number.

**22.** 15 is the difference between one-fourth of a number and 12.

**23.** Three times a number, increased by 20 is greater than 56.

**24.** 14 more than, 10 times a number is the same as the difference between 100 and 4 times the number.

**25.** The sum of 5 times a number and 30 is less than or equal to 50.

# Unit I – The Structure of Mathematics

*Part B – Further Investigation of Number Symbols*

## LESSON 1 The Development of Our Number System

**Objective:** To identify the different types of numbers and understand how they are related to each other.

**Important Terms:**

**Well-Defined Operation** – an operation which satisfies the conditions of ***Existence*** (must get an answer), ***Uniqueness*** (must get only one answer for each number combination), and ***Closure*** (answer must be in the set we are working in).

**Natural Numbers** – the numbers we use to count objects. The complete set of natural numbers is shown as {1, 2, 3, ...}, usually called "the set N." These numbers are also called counting numbers.

**Whole Numbers** – the natural numbers combined with the number 0. The complete set of whole numbers is shown as {0, 1, 2, 3, ...} and usually called "the set W."

**Integers** – the whole numbers combined with all of their opposites. The complete set is shown as {..., ⁻3, ⁻2, ⁻1, 0, 1, 2, 3, ...}.