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Unit I – The Structure of Mathematics

Part A – Mathematics as a Language

p. 2 Lesson 1 – Mathematical Parts of Speech

Part of Speech	Meaning
1. Placeholder symbol	Holds the place of a number until number is identified
2. Grouping symbol	Brackets
3. Number symbol	Three and, forty-five hundredths repeating
4. Operation symbol	Add
5. Relation symbol	Is equal to
6. Number symbol	Sixty-three thousand, two hundred thirty one
7. Relation symbol	Is greater than or equal to
8. Number symbol	Approximately 2.2
9. Operation symbol	Subtract
10. Grouping symbol	Braces
11. Number symbol	10,000
12. Operation symbol	Divide
13. Relation symbol	Is not less than
14. Grouping symbol	Parentheses
15. Number symbol	Two-thirds
16. Number symbol	Forty-one and seven hundred sixty-five thousandths
17. Operation symbol	Multiply
18. Placeholder symbol	Holds the place of a number until number is identified
19. Relation symbol	Is not equal to
20. Operation symbol	Multiply

pp. 4, 5 Lesson 2 – Mathematical Expressions

1. Closed phrase	2. Open phrase	3. Open sentence	4. Closed sentence
5. Closed sentence	6. Open phrase	7. Open phrase	8. Closed sentence
9. Open sentence	10. Closed phrase	13. $5 - z = 2$ {4, 5, 6}	14. $8 - 2^3 = 4 - 2$
11. $6 + 8(3)$ $6 + 24$ 30	12. $m + 2$ {0, 1, 2} $0 + 2 = 2$ $1 + 2 = 3$ $2 + 2 = 4$ Range = {2, 3, 4}	$5 - 4 = 2$ $1 = 2$ F $5 - 5 = 2$ $0 = 2$ F $5 - 6 = 2$ $-1 = 2$ F Solution set is { }	$8 - 8 = 4 - 2$ $0 = 2$ False
15. $\frac{4z^3}{3} > 2 \cdot 2$ $\frac{4}{3} > 4$ $4 > 4$ False	16. $w^2 - w$ {0, 1, 2} $0^2 - 0 = 0$ $1^2 - 1 = 0$ $2^2 - 2 = 2$ Range = {0, 2}	17. $9y$ {0, 1, 2} $9 \cdot 0 = 0$ $9 \cdot 1 = 9$ $9 \cdot 2 = 18$ Range = {0, 9, 18}	18. $\frac{15+18}{3} \neq 5 + 6$ $\frac{33}{3} \neq 5 + 6$ $11 \neq 11$ False
19. $3k + 11 < 17$ {4, 5, 6} $3 \cdot 4 + 11 < 17$ $12 + 11 < 17$ $23 < 17$ F $3 \cdot 5 + 11 < 17$ $15 + 11 < 17$ $26 < 17$ F $3 \cdot 6 + 11 < 17$ $18 + 11 < 17$ $29 < 17$ F Solution set is { }	20. $2 \cdot 4 - 1$ $8 - 1$ 7		

pp. 6, 7 Lesson 3 – Translation of Mathematical Symbols

1. $(q + 6) + 4$	2. $m \cdot n + 11$ $\frac{m+n}{11}$ $(m \times n) \div 11$	3. $(x + y) + 27$	4. $p - 8$ or $8 - p$
5. $(a + b) \div 6$ $\frac{a+b}{6}$	6. $a^4 + 12$	7. $10 \cdot z - 8 \cdot x$ $(10z) - (8x)$	$10z - 8x$
8. $y \cdot 6 + 5 \cdot z$ $6y + 5z$	9. $5 \cdot y - y^3$ $5y - y^3$	10. $(x + 5)(x - 8)$	

11. $(y-9)^2 + (4 \cdot t) \frac{(y-9)^2}{4t}$ 12. $(q+r)-(c-d)$ 13. $(6+b)+(7+c)$ 14. $(v^3-w^3)+e^2 \frac{v^3-w^3}{e^2}$
 15. $h^2+(j^3-b)$ 16. $9+4n=55$ 17. $8n-20=4n+20$ 18. $2n=28$
 19. $6n-12=3n+8$ 20. $150=6n-30$ 21. $25-4n < 3n+100$ 22. $15 = \frac{1}{4}n-12 \quad 15 = \frac{7}{4}-12$
 23. $3n+20 > 56$ 24. $10n+14=100-4n$ 25. $5n+30 \leq 50$

Part B – Further Investigation of Number Symbols

p. 9 Lesson 1 – The Development of Our Number System

- Yes – (answers will vary) adding natural numbers gives natural numbers
 - No – (answers will vary) $2-5 = -3$ and -3 is not a natural number
 - Yes – (answers will vary) multiplying natural numbers gives natural numbers
 - No – (answers will vary) $2+5 = \frac{7}{2}$ and $\frac{7}{2}$ is not a natural number
- No – (answers will vary) $1+3 = 4$ and 4 is not an odd integer
 - No – (answers will vary) $3-1 = 2$ and 2 is not an odd integer
 - Yes – (answers will vary) multiplying odd integers gives odd integers
 - No – (answers will vary) $5+0$ does not give an answer
- Yes – (answers will vary) adding halves gives halves
 - Yes – (answers will vary) subtracting halves gives halves
 - No – (answers will vary) $\frac{1}{2} \cdot \frac{2}{3} = \frac{2}{6}$ and $\frac{2}{6}$ is not in the set of halves
 - No – (answers will vary) $\frac{1}{2} + \frac{2}{3}$ does not give an answer
- Yes – (answers will vary) adding non-negative multiples of 5 gives non-negative multiples of 5
 - No – (answers will vary) $5-10 = -5$ and -5 is not a non-negative multiple of 5
 - Yes – (answers will vary) multiplying non-negative multiples of 5 gives non-negative multiples of 5
 - No – (answers will vary) $10+5 = 2$ and 2 is not a non-negative multiple of 5
- $\{6, -7, \frac{1}{2}, -100, -3, 47, 2, 0, \frac{1}{4}, -\frac{1}{2}\}$
 - $\{6, \frac{1}{2}, 47, 2, \frac{1}{4}\}$
 - All are rational numbers.
 - $\{6, \frac{1}{2}, 47, 2, 0, \frac{1}{4}\}$

p. 11 Lesson 2 – Fraction Forms and Decimal Forms

- $\frac{3}{4} = \frac{3}{4} \cdot 1 = \frac{3}{4} \cdot \frac{5}{5} = \frac{15}{20}$
- $\frac{7}{10} = \frac{7}{10} \cdot 1 = \frac{7}{10} \cdot \frac{6}{6} = \frac{42}{60}$
- $\frac{8}{8} = \frac{8}{8} \cdot 1 = \frac{8}{8} \cdot \frac{10}{10} = \frac{80}{80}$
- $\frac{3}{16} = \frac{3}{16} \cdot 1 = \frac{3}{16} \cdot \frac{2}{2} = \frac{6}{32}$
- $\frac{1}{2} = \frac{1}{2} \cdot 1 = \frac{1}{2} \cdot \frac{12}{12} = \frac{12}{24}$
- $\frac{8}{11} = \frac{8}{11} \cdot 1 = \frac{8}{11} \cdot \frac{3}{3} = \frac{24}{33}$
- $\frac{0}{6} = \frac{0}{6} \cdot 1 = \frac{0}{6} \cdot \frac{12}{12} = \frac{0}{72}$
- $\frac{10}{10} = \frac{10}{10} \cdot 1 = \frac{10}{10} \cdot \frac{8}{8} = \frac{80}{80}$
- $\frac{8}{3} = \frac{8}{3} \cdot 1 = \frac{8}{3} \cdot \frac{15}{15} = \frac{120}{45}$
- $\frac{3 \cdot 11}{8 \cdot 11} = \frac{3}{8}$
- $\frac{2 \cdot 9}{3 \cdot 9} = \frac{2}{3}$
- $\frac{4 \cdot 1}{30 \cdot 1} = \frac{4}{30}$ (41 is prime)
- $\frac{6 \cdot 3}{13 \cdot 3} = \frac{6}{13}$
- $\frac{2 \cdot 3}{22 \cdot 3} = \frac{2}{22}$
- $\frac{1 \cdot 7}{3 \cdot 7} = \frac{1}{3}$
- $\frac{2 \cdot 8}{3 \cdot 8} = \frac{2}{3}$
- $\frac{13 \cdot 1}{13 \cdot 1} = \frac{13}{13}$ (13 is prime)
- $\frac{4 \cdot 6}{5 \cdot 6} = \frac{4}{5}$
- $\frac{4 \cdot 10}{7 \cdot 10} = \frac{4}{7}$

p. 14 Lesson 3 – Changing Fraction Forms to Decimal Forms

- $\frac{1}{4} = 4 \overline{)1.00} \quad .25$
- $\frac{7}{20} = 20 \overline{)7.00} \quad .35$
- $\frac{12}{30} = 30 \overline{)19.00} \quad .38$
- $\frac{15}{2} = 2 \overline{)15.0} \quad 7.5$
- $\frac{4}{3} = 3 \overline{)4.0} \quad .8$
- $\frac{11}{40} = 40 \overline{)11.000} \quad .275$
- $\frac{37}{8} = 8 \overline{)37.000} \quad 4.625$
- $\frac{3}{16} = 16 \overline{)3.0000} \quad .1875$