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Instructional Aids

Detailed Solutions for Progress Tests

Unit I – The Structure of Mathematics

Part A – Mathematics as a Language

Part B – Further Investigation of Number Symbols

ALGEBRA - A COMPLETE COURSE

STRUCTURE OF MATHEMATICS (Unit I)

TAPE 1

A. Mathematics as a Language
- Parts of Speech
- Expressions
- Translations

B. More on Numbers
- Number System
- Fraction & Decimal Forms)
- Equivalent Forms - 1
- Equivalent Forms - 2
- Percent
- Primes and Composites
- Least Common Multiple
- Greatest Common Factor

TAPE 2

C. More on Operations
- Order
- Properties - 1
- Properties - 2
- Fractions - 1
- Fractions - 2
- Fractions - 3
- Decimals
- Signed Numbers - 1
- Signed Numbers - 2
- Signed Numbers - 3
- Signed Numbers - 4

TAPE 3

D. More on Relations
- Order
- Properties - 1
- Properties - 2

E. Models
- Sets
- Functions

FIRST DEGREE RELATIONS

One Placeholder (Unit II)

TAPE 4

A. Basic Relations
- Solutions
- Making Zeros
- Making Ones
- Combinations

B. Complications
- Grouping Symbols
- Like Terms on the Same Side
- Placeholders on Both sides
- Combinations

C. Special Cases
- No Solution
- Infinite Solutions

D. Systems
- "And"
- "Or"
- Absolute Value - 1
- Absolute Value - 2
- Absolute Value - 3

TAPE 5

E. Problem Solving
- Setup
- "Numbers"
- "Consecutive Integers"
- "Age"
- "Geometric Figures"
- "Motion"
- "Percent"

Two Placeholders (Unit III)

TAPE 6

A. Solution Sets
- Equations
- Inequalities
- Graphing Terms
- $y = mx$
- $y = mx + b$
- Intercepts

B. Special Cases
- $y = , >, < a$
- $x = , >, < a$
- Absolute Value

TAPE 7

C. Relations from Solutions
- Given Slope and y-intercept
- Given Slope and One Solution
- Given Two Solutions
- Special Cases

D. Systems
- Graphic - 1
- Graphic - 2
- Algebraic - 1
- Algebraic - 2

TAPE 8

E. Special Cases
- No solution
- Infinite Solutions

F. Problem Solving
- Setup
- "Numbers"
- "Age"
- "Geometric Figures"
- "Motion"
- "Percent"
- "Mixture"

Three Placeholders (Unit IV)

TAPE 9

A. Solution Sets
- One Sentence
- Two Sentences
- Three or More Sentences

B. Special Cases
- No Solution
- Infinite Solutions

C. Problem Solving
- "Numbers"
- "Age"
- "Geometric Figures"
- "Mixture"

SECOND DEGREE RELATIONS (or higher)

Polynomials (Unit V)

TAPE 10

A. Exponent Notation
- Terminology
- Operations - 1
- Operations - 2
- Special Cases
- Scientific Notation

B. Polynomials
- Terminology - 1
- Terminology - 2
- Operations - 1
- Operations - 2
- Operations - 3

TAPE 11

C. Solving Relations
- Principle of Zero Products
- Common Factor
- Difference of Squares
- Perfect Square Trinomial
- General Trinomial
- Four or More Terms
- Sum or Difference of Cubes
- General Strategy
- Synthetic Division
- Literal Equations

TAPE 12

D. Problem Solving
- "Numbers"
- "Consecutive Integers"
- "Geometric Figures"
- "Formula"

Algebraic Fractions (Unit VI)

TAPE 13

A. Operations
- Simplifying
- Multiplying
- Dividing
- Adding or Subtracting
- Complex

B. Solving Relations
- Equations - 1
- Equations - 2
- Inequalities
- Literal Equations

C. Problem Solving
- "Fractions"
- "Work"
- "Motion"
- "Direct Variation"
- "Inverse Variation"
- "Mixed Variation"

RATIONAL DEGREE RELATIONS (Unit VII)

TAPE 14

A. Rational Exponents
- Radicals
- "k^{th}" roots

B. Operations
- Multiplying
- Simplifying
- Dividing
- Adding or Subtracting
- In Polynomials
- Rationalizing

C. Solving Relations
- One Radical
- Two Radicals

D. Problem Solving
- "Distance"
- "Formula"

E. Complex Numbers
- Definition
- Adding or Subtracting
- Multiplying
- Dividing

SUMMARY OF SECOND DEGREE RELATIONS*

Quadratic (Unit VIII)

TAPE 15

A. Solving from Standard Form
- $a, b, \text{or } c = 0$
- $a, b, c \neq 0$
- Quadratic Formula
- Checking Solutions
- Inequalities (0:05:33)

B. Relations Quadratic In Form
- Higher Order to Lower Order > 0
- Lower Order to Higher Order > 0
- Integer Order < 0

C. Problem Solving
- "Geometric Figures"
- "Pythagorean Theorem"
- "Work"
- "Motion"

Conic Sections (Unit IX)

TAPE 16

A. Parabolas
- Origins
- Standard Form - 1
- Standard Form - 2
- Standard Form - 3
- Standard Form - 4
- Intercepts

B. Circles
- Standard Form
- Non-standard Form

C. Ellipses
- Standard Form
- Non-standard Form

D. Hyperbolas
- Standard Form
- Non-standard Form

E. Systems
- One 1st Degree and One 2nd Degree
- Two 2nd Degree

F. Problem Solving
- "Numbers"
- "Geometric Figures"

LITERAL DEGREE RELATIONS (Unit X)

TAPE 17

A. Exponential Functions
- Functions of x
- Functions of y

B. Logarithmic Functions
- Translating
- Graphs of Solution Sets

C. Operations
- Properties
- Finding Logarithms)
- Computation

D. Solving Equations
- Exponential
- Logarithmic

* This would logically follow ALGEBRAIC FRACTIONS (Unit VI) except that a knowledge of roots and radicals is necessary for a comprehensive summary.

Quiz Form A

Name _____

Class _____ **Date** _____ **Score** _____

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language

Lesson 1 – Mathematical Parts of Speech

Tell what part of mathematical speech each of the following is.

1.	( )	Grouping Symbol
2.	m	Placeholder Symbol
3.	$\neq$	Relation Symbol
4.	$\div$	Operation Symbol
5.	$\square$	Placeholder Symbol
6.	88	Number Symbol
7.	$\cdot$	Operation Symbol
8.	[ ]	Grouping Symbol
9.	$>$	Relation Symbol
10.	18.7	Number Symbol

Quiz Form B

Name _____

Class _____ **Date** _____ **Score** _____

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language
Lesson 1 – Mathematical Parts of Speech

Tell what part of mathematical speech each of the following is.

1.	101.01	Number Symbol
2.	$<$	Relation Symbol
3.	{ }	Grouping Symbol
4.	$\div$	Operation Symbol
5.	13	Number Symbol
6.	$\triangle$	Placeholder Symbol
7.	x	Placeholder Symbol
8.	$\not>$	Relation Symbol
9.	$-$	Operation Symbol
10.	( )	Grouping Symbol

Quiz Form A

Name _____

Class _____ **Date** _____ **Score** _____

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language
Lesson 2 – Mathematical Expressions

Tell whether each of the following expressions is an open phrase, closed phrase, open sentence, or closed sentence.

1. $5-3=6\cdot 2$ closed sentence (relation symbol, but no placeholder)

2. $9x+4$ open phrase (placeholder, but no relation symbol)

3. $\sqrt{y+4}$ open phrase (placeholder, but no relation symbol)

4. $x-6=3$ open sentence (placeholder and relation symbol)

5. $6y+2=50$ open sentence (placeholder and relation symbol)

6. $9-6=3$ closed sentence (relation symbol, but no placeholder)

7. $3\cdot 7+6$ closed phrase (no placeholder and no relation symbol)

8. $(9-5)8$ closed phrase (no placeholder and no relation symbol)

Name _____

Unit I, Part A, Lesson 2, Quiz Form A
– Continued –

In the following exercises, take the appropriate mathematical action with each expression. If you need a domain, use {4, 5, 6}. If you need a replacement set, use {10, 11, 12}.

9. $2x + x$ Domain $\{4, 5, 6\}$ Range $\{12, 15, 18\}$

Using domain $\{4, 5, 6\}$, we substitute and evaluate:

$2(4) + (4) \rightarrow 8 + 4 \rightarrow 12$

$2(5) + (5) \rightarrow 10 + 5 \rightarrow 15$

$2(6) + (6) \rightarrow 12 + 6 \rightarrow 18$

So the range is $\{12, 15, 18\}$

10. $5x + 9 = 59$ $\{10\}$

Substitute from $\{10, 11, 12\}$, and obtain a solution set of values that make the expression true.

$5(10) + 9 = 59$ T

$5(11) + 9 = 59$ F

$5(12) + 9 = 59$ F

So our solution set is $\{10\}$

11. $\frac{13+9}{2} > 5 \cdot 3$ False

Closed sentence, so tell if it is true or false.

$\frac{13+9}{2} > 5 \cdot 3$

$\frac{22}{2} > 15$

$11 > 15$ F

12. $\frac{x}{2} < 13$ $\{10, 11, 12\}$

Substitute from $\{10, 11, 12\}$, and obtain a solution set of values that make the expression true.

$\frac{10}{2} < 13 \rightarrow 5 < 13$ T

$\frac{11}{2} < 13 \rightarrow 5\frac{1}{2} < 13$ T

$\frac{12}{2} < 13 \rightarrow 6 < 13$ T

So our solution set is $\{10, 11, 12\}$.

Quiz Form B

Name _____

Class _____ **Date** _____ **Score** _____

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language
Lesson 2 – Mathematical Expressions

Tell whether each of the following expressions is an open phrase, closed phrase, open sentence, or closed sentence.

1. $(7+2)\cdot 3$ closed phrase (no placeholder and no relation symbol)

2. $6\cdot 2 = 7+5$ closed sentence (relation symbol, but no placeholder)

3. $y+4=11$ open sentence (placeholder and relation symbol)

4. $6x+2=14$ open sentence (placeholder and relation symbol)

5. $7-2\cdot 3$ closed phrase (no placeholder and no relation symbol)

6. $2y-8$ open phrase (placeholder, but no relation symbol)

7. $\sqrt{y-3}$ open phrase (placeholder, but no relation symbol)

8. $2+4=7$ closed sentence (relation symbol, but no placeholder)

Name _____

Unit I, Part A, Lesson 2, Quiz Form B
– Continued –

In the following exercises, take the appropriate mathematical action with each expression. If you need a domain, use {4, 5, 6}. If you need a replacement set, use {10, 11, 12}.

9. $\frac{x}{2} > 3$ $\{10, 11, 12\}$

Substitute from $\{10, 11, 12\}$, and obtain a solution set of values that make the expression true.

$\frac{10}{2} > 3 \rightarrow 5 > 3$ T

$\frac{11}{2} > 3 \rightarrow 5\frac{1}{2} > 3$ T

$\frac{12}{2} > 3 \rightarrow 6 > 3$ T

So our solution set is $\{10, 11, 12\}$

10. $3x - 2$ Domain $\{4, 5, 6\}$ Range $\{10, 13, 16\}$

Using domain $\{4, 5, 6\}$, we substitute and evaluate:

$3(4) - 2 \rightarrow 12 - 2 \rightarrow 10$

$3(5) - 2 \rightarrow 15 - 2 \rightarrow 13$

$3(6) - 2 \rightarrow 18 - 2 \rightarrow 16$

So the range is $\{10, 13, 16\}$

11. $4x - 3 = 41$ $\{11\}$

Substitute from $\{10, 11, 12\}$, and obtain a solution set of values that make the expression true.

$4(10) - 3 = 41 \rightarrow 40 - 3 = 41$ F

$4(11) - 3 = 41 \rightarrow 44 - 3 = 41$ T

$4(12) - 3 = 41 \rightarrow 48 - 3 = 41$ F

So our solution set is $\{11\}$.

12. $6 \div 2 < \frac{10 - 5}{5}$ False

Closed sentence, so tell if it is true or false.

$6 \div 2 < \frac{10-5}{5}$

$3 < \frac{5}{5}$

$3 < 1$ F

Quiz Form A

Name _____

Class _____ **Date** _____ **Score** _____

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language
Lesson 3 – Translation of Mathematical Symbols

Translate the following English phrases into expressions with mathematical symbols.

1. x divided by 7, decreased by 2

$\frac{x}{7} - 2$ or $(x \div 7) - 2$

2. the sum of x and y

$x + y$

3. 15 less than the total of x and y

$(x + y) - 15$

4. the difference between a and b, multiplied by the product of x and y

$(x \cdot y)(a - b)$

Translate the following English sentences into expressions with mathematical symbols.

5. The product of 4 and a number is 20.

$4m = 20$

6. 14 plus, 3 times a number is 59.

$14 + 3a = 59$

7. 25 is less than the quotient of 9 and x.

$25 < \frac{9}{x}$ or $25 < 9 \div x$

8. The sum of 20 and the product of 9 and x is greater than 110.

$20 + (9 \cdot x) > 110$

Quiz Form B

Name _____

Class _____ **Date** _____ **Score** _____

Unit I – The Structure of Mathematics
Part A – Mathematics as a Language
Lesson 3 – Translation of Mathematical Symbols

Translate the following English phrases into expressions with mathematical symbols.

1. 12 more than the difference of x and y

$(x-y)+12$

2. the quotient of 7 and 3

$\frac{7}{3}$ or $7 \div 3$

3. 32 greater than the sum of a and b

$(a+b)+32$

4. the sum of m and n, multiplied by the quotient of 1 and 2

$(m+n)\cdot \frac{1}{2}$

Translate the following English sentences into expressions with mathematical symbols.

5. 17 is the difference between 63 and a number.

$17 = 63 - n$

6. 34 is greater than the sum of 7 and y.

$34 > 7+y$

7. 81 minus, 4 times a number is 21.

$81-4x = 21$

8. The difference of 10 and a number is less than 52.

$10-n < 52$