

Table of Contents

Unit I – The Structure of Mathematics

Part A – Mathematics as a Language

LESSON 1	Mathematical Parts of Speech	1
LESSON 2	Mathematical Expressions	2
LESSON 3	Translation of Mathematical Symbols	3

Part B – Further Investigation of Number Symbols

LESSON 1	The Development of Our Number System	6
LESSON 2	Fraction Forms and Decimal Forms	9
LESSON 3	Changing Fraction Forms to Decimal Forms	11
LESSON 4	Changing Decimal Forms to Fraction Forms	12
LESSON 5	Percent	13
LESSON 6	Primes, Composites, and Factoring	14
LESSON 7	Least Common Multiple	16
LESSON 8	Greatest Common Factor	17

Part C – Further Investigation of Operation Symbols

LESSON 1	Order of Operations	18
LESSON 2	Properties of Operations	19
LESSON 3	Properties of Operations with Special Numbers	20
LESSON 4	Operations with Fractions – Multiplication	22
LESSON 5	Operations with Fractions – Addition and Subtraction	23
LESSON 6	Operations with Fractions – Division	25
LESSON 7	Operations with Decimals	26
LESSON 8	Operations with Signed Numbers – Vectors and Absolute Value	28
LESSON 9	Operations with Signed Numbers – Addition	30
LESSON 10	Operations with Signed Numbers – Subtraction	32
LESSON 11	Operations with Signed Numbers – Multiplication and Division	33

Part D – Further Investigation of Relation Symbols

LESSON 1	Order of Numbers and the Number Line	36
LESSON 2	Properties of Equality	37
LESSON 3	Properties of Inequality	38

Part E – Mathematical Models

LESSON 1	The Mathematics of Sets	40
LESSON 2	The Mathematics of Functions	44

5 PARTS OF MATHEMATICAL SPEECH

1. Number Symbols
“things”
(nouns) $\{1, 2, 3, \frac{4}{5}, .9, \dots\}$
2. Operation Symbols
“actions”
(verbs) $\{+, -, \times, \div, \dots\}$
3. Relation Symbols
“comparisons” $\{=, >, <, \dots\}$
4. Grouping Symbols
“associations”
(punctuation) $\{(), [], \{ \}, \dots\}$
5. Placeholder Symbols
“hold the place”
(pronouns) $\{a, b, c, ?, \square, \dots\}$

TYPES OF MATHEMATICAL EXPRESSIONS

1. Closed Phrase

Ex. $7 + 9$

no relation
no placeholder

So, Evaluate

Ex. 16

2. Open Phrase

Ex. $7 + n$

no relation
has placeholder

So, Substitute and Evaluate

Ex. $\begin{cases} \text{Domain} = \{0, 1, 2\} \\ \text{Range} = \{7, 8, 9\} \end{cases}$

3. Closed Sentence

Ex. $7 + 9 = 17$

has relation
no placeholder

So, True or False

Ex. False

4. Open Sentence

Ex. $7 + n = 17$

has relation
has placeholder

So, Substitute and
True or False

Ex. $\begin{cases} \text{Replacement} = \{4, 5, 6\} \\ \text{Solution} = \{ \} \end{cases}$

TRANSLATION OF MATH SYMBOLS

1. Number Symbols {1, 2, 3, $\frac{4}{5}$, .9, ...}

3 → three

2. Operation Symbols {+, −, ×, ÷, ...}

+	add	rise	increase
	sum	climb	combine
	plus	total	number more than
	gain		

−	subtract	loss	take away
	difference	fall	deduct
	minus	decrease	number less than

x or ·	times	of	twice
	multiply	double	
	product	triple	

÷	divide	goes into
	quotient	fraction

3. Relation Symbols {=, >, <, ...}

= is equal to, is the same as

> is greater than, is more than

< is less than, is smaller than

TRANSLATION OF MATH SYMBOLS

4. Grouping Symbols $\{(), [], \{ }, \dots\}$

() parentheses

[] brackets

{ } braces

the quantity

“sum”

“difference”

“product”

“quotient”

5. Placeholder Symbols $\{a, b, c, \dots, ?, \dots, \square, \dots\}$

a “number”

the “unknown”

the “age”

the “distance”

the “weight”

⋮

TRANSLATION OF MATH SYMBOLS

The sum of a number and 7, is equal to, 20.

$$n + 7 = 20$$

4 is 1 less than the quotient of a number and 9.

$$4 = \frac{n}{9} - 1$$

39 is greater than the result of multiplying the sum of 6 and some number by 3.

$$39 > (6 + n) \cdot 3$$